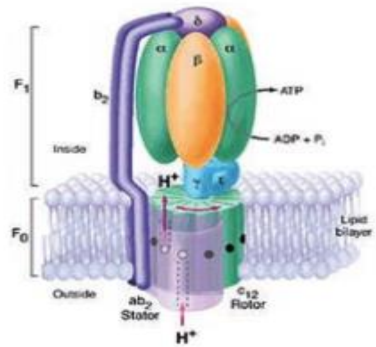
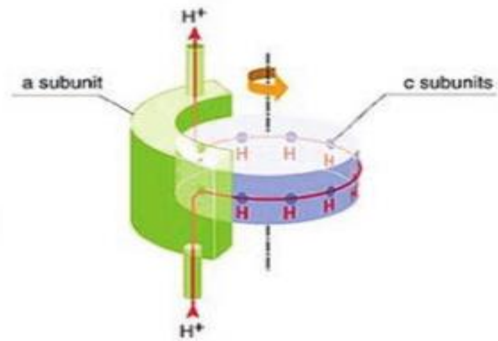
An Introduction to Vayenas Elementary Particles

[Protons in Catalysis on Earth & the Universe]

BIOLOGY

ATP SYNTHESIS

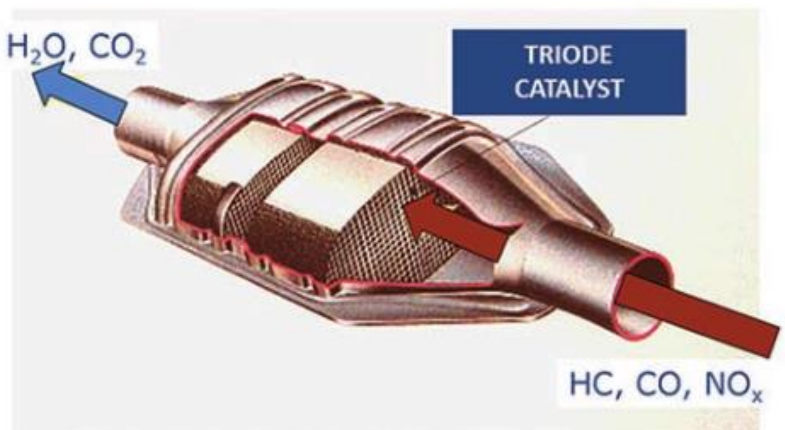




■ **Biology: Enzymes – ATP synthesis/Protons in Rotational Catalysis**

Energy ~ 0.1 – 1 eV ; Wavelength ~ 0.1 – 1 nm

CHEMISTRY



■ **Chemistry: Catalysis and Fuel Cells/Protons accelerators in chemical & energy production**

Energy ~ 1 eV ; Wavelength ~ 0.1 nm

PHYSICS ?

■ **Physics: Structure of Protons**

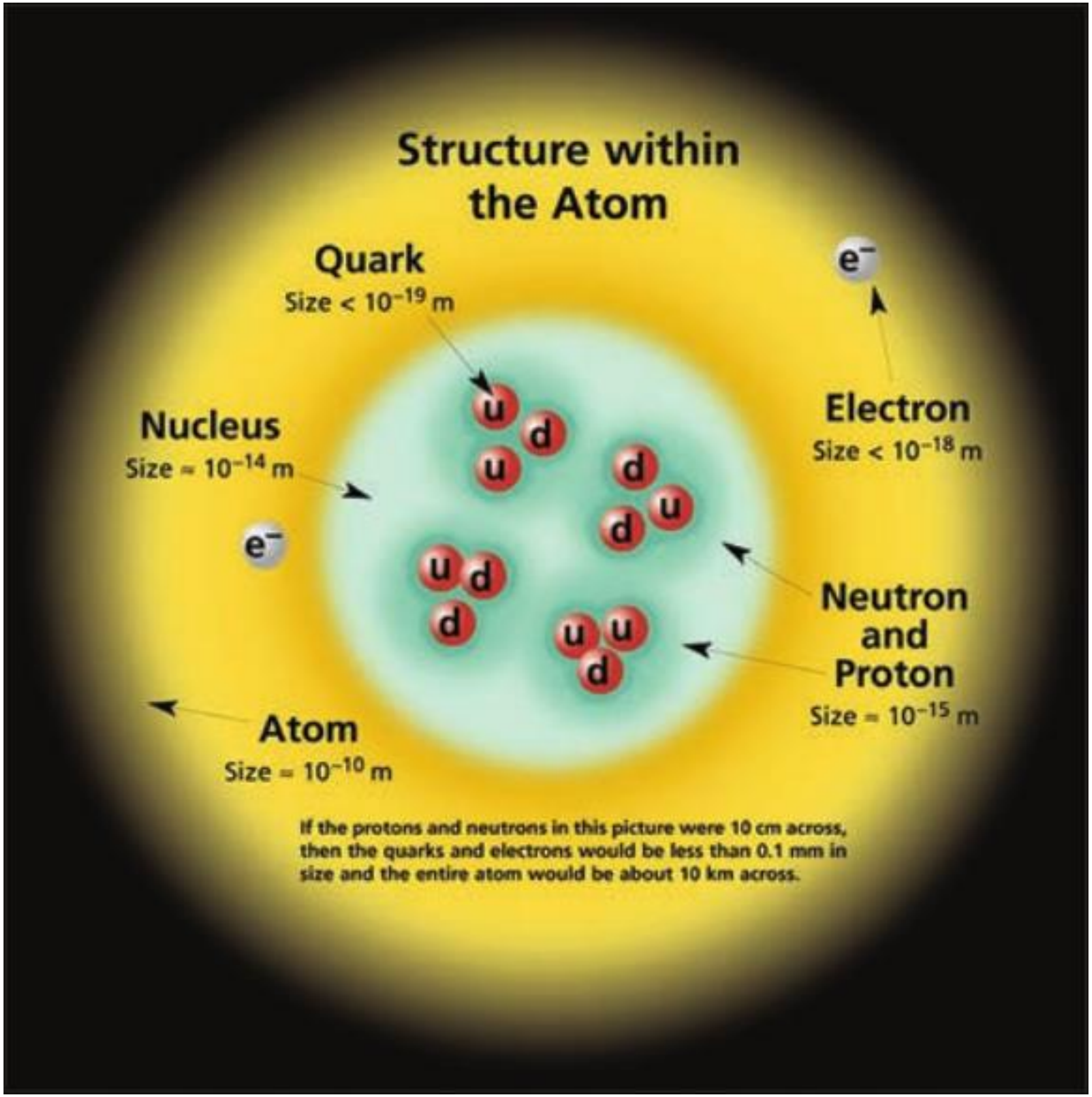
Energy ~ 1 GeV ; Wavelength ~ 1 fm (de Broglie)

■ Thermodynamics of Key Reactions

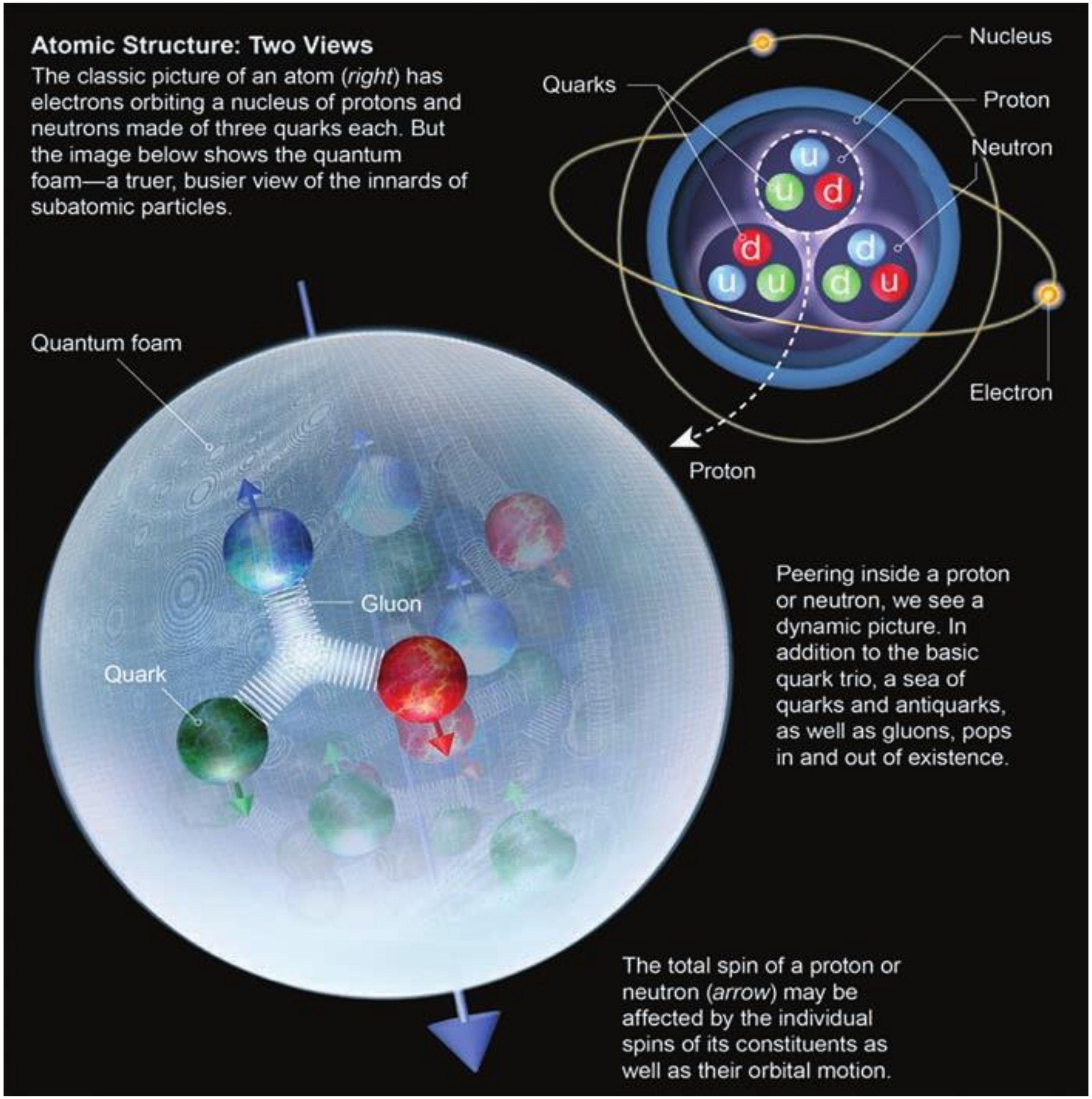
	ΔH		ΔS	$T_{cr}=\Delta H/\Delta S$	$T_{ad} = -\Delta H/C_p$
	kJ/mol	eV/atom	J/mol-K	K	K
$\frac{1}{2} N_2 + 3/2\ H_2 \rightarrow NH_3$	-45.85	-0.48	-99.1	463	1565
HUMAN SURVIVAL					
$H_2 + \frac{1}{2} O_2 \rightarrow H_2O$	-241.8	-2.51	-44.5	5433	8255
BIOLOGICAL EXISTENCE					
$p + e^- \rightarrow H$	-1312	-13.6	-5.81	22600	44800
CREATION OF ATOMS AND MOLECULES					
$4p \rightarrow {}^4He + 2e^+ + 2\nu_e$	$-2.57 \cdot 10^9$	$-2.67 \cdot 10^7$	-9.19	$2.81 \cdot 10^{11}$	$8.8 \cdot 10^{10}$
HYDROGEN FUSION					
"Quark-gluon plasma condensation" = Baryogenesis					
$3\nu_c + e^+ \rightarrow p^+$	$-6.02 \cdot 10^{10}$	$-.625 \cdot 10^9$	-11.6	$5.19 \cdot 10^{12}$	$2.05 \cdot 10^{12}$
CREATION OF VISIBLE MATTER					

ALL ARE EXOTHERMIC WITH COMPARABLE ΔS

■ Structure of the Nucleus according to Standard Model (SM)



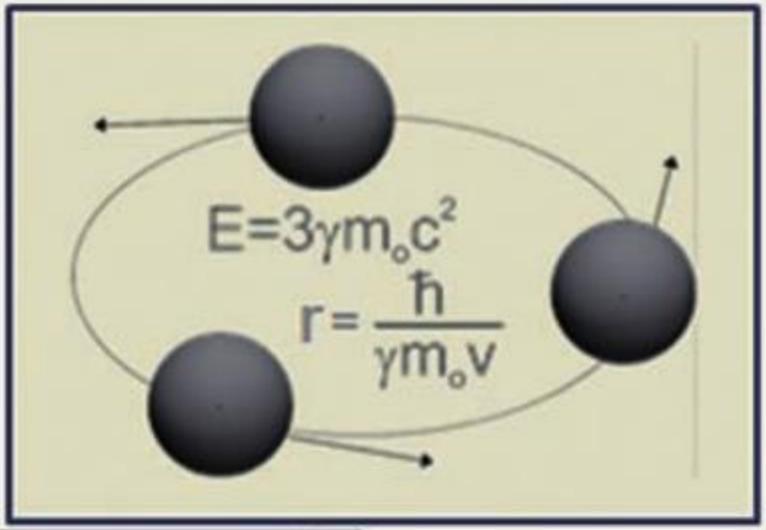
■ Proton Structure: Quarks & Gluons



■ Vayenas RLM Model: $m_g = \gamma^3 m_0$

- *Governing Equations*

Attractive force = Gravity



A: Neutrino's corpuscular nature

- Newton-Einstein equation for cycling motion:

$$F = \gamma m_o \frac{v^2}{r}$$

- Newton's gravitational law: m_g: gravitational mass

$$F = G \frac{m_g^2}{\sqrt{3} r^2}$$

B: Neutrino's ondular nature

de Broglie :

$$r = \lambda\!\!\!^{-} = \frac{\hbar}{\gamma m_o v}$$

Solution of Eqs

Particle

$$\frac{\gamma m_o \mathrm{v}^2}{r} = \frac{G m_o^2 \gamma^6}{\sqrt{3}\, r^2} \Leftrightarrow r = \frac{G m_o}{\sqrt{3}\, c^2}\gamma^5 \left(\frac{\gamma^2}{\gamma^2 - 1}\right)$$

Wave

$$r = \frac{\hbar}{\gamma m_o \mathrm{v}} = \frac{3\hbar}{m \mathrm{v}}$$

Results

$v \approx c$; $\gamma = 7.163 \cdot 10^9$	For $m_o = 0.05$ eV/c^2 $m = 1000$ MeV/c^2
$m = 3\gamma m_o = 3^{13/12} m_o^{2/3} m_{Pl}^{1/3}$	For $m_o = 0.0437$ eV/c^2 $m = 939.565$ MeV/c^2!
$m_{Pl} = (\hbar c / G)^{1/2}$ Planck mass	Neutrino mass Neutron mass

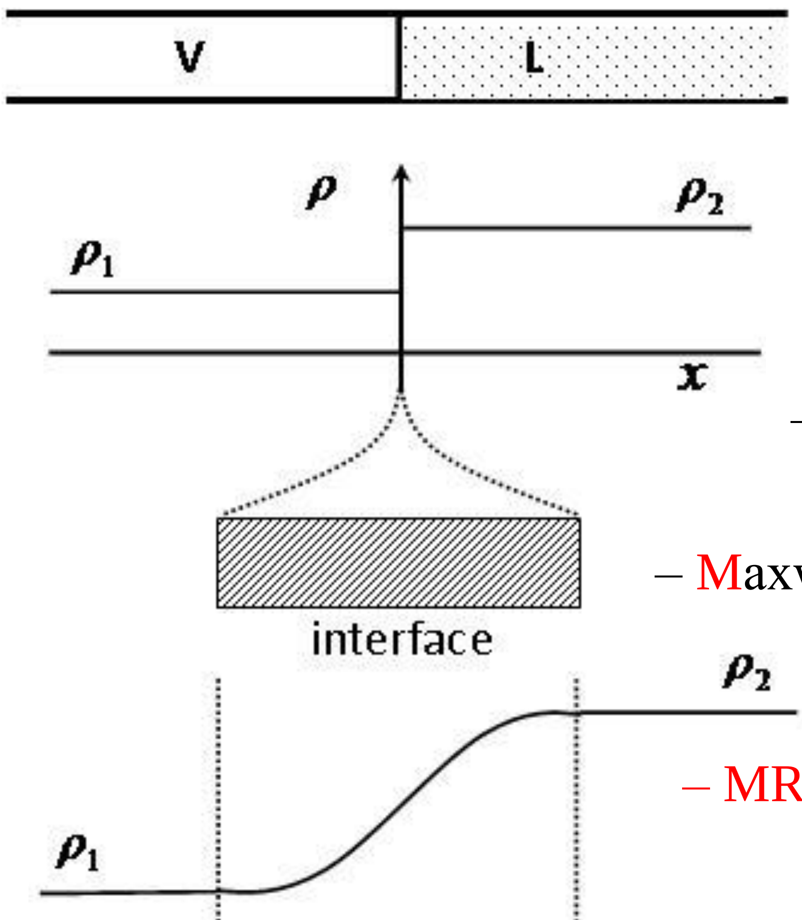
- Vayenas CG, Souentie S & Fokas A (2013) A Bohr-type model with gravity as the attractive force. arXiv:1306.5979v1 [physics.gen-ph]; (2014) Physica A 405:360-379.

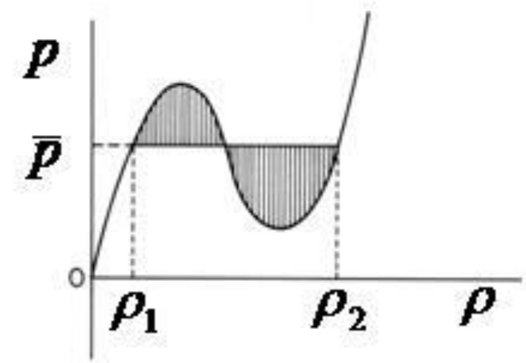
Our Starting Point: The Simplest Intefrace

- ***Revisit Van der Waals/Maxwell's Thermodynamics***

$$\mathrm{div}\mathbf{T} = \mathbf{0}$$

$$\mathbf{T} = \left[-p(\rho) + \alpha \nabla^2 \rho + \beta |\nabla \rho|^2 \right] \mathbf{1} + \gamma \, \mathrm{grad}^2 \rho + \delta \, \nabla \rho \otimes \nabla \rho$$





– Equilibrium : $p(\rho_1) = p(\rho_2) = \bar{p}$

– Maxwell's Rule (MR): $\int_{\rho_1}^{\rho_2} [p(\rho) - \bar{p}] \frac{d\rho}{\rho^2} = 0$

– MR Modification: $1/\rho^2 \rightarrow E(\rho) = \frac{1}{a} \exp\left(2 \int \frac{b}{a} d\rho \right)$

$$a = \alpha + \gamma; \quad b = \beta + \delta$$

- ***Solution Details – Planar Interfaces***

  - *One Dimension*

$$\rho = \rho(x) \quad \Rightarrow \quad \begin{cases} T_{xx} = T = -p(\rho) + a\rho_{xx} + b\rho_x^2 \\ T_{yy} = T_{zz} = -p(\rho) + \alpha\rho_{xx} + \beta\rho_x^2 \end{cases}$$

$$\partial \mathrm{T}/\partial x = 0 \qquad a\rho_{xx} + b\rho_x^2 = p(\rho) - \bar{p}\,; \qquad \begin{cases} a \equiv \alpha + \gamma \\ b \equiv \beta + \delta \end{cases}$$

  - *Analytical Solutions/Conditions for Existence*

$$p(\rho_1) = p(\rho_2) = \bar{p}\,, \quad \int_{\rho_1}^{\rho_2} [p(\rho) - \bar{p}]\,\mathrm{E}(\rho)\,d\rho = 0\,; \quad \mathrm{E}(\rho) \equiv \frac{1}{a}\exp\left(2\int \frac{b}{a}\,d\rho\right)$$

$$x = x_0 + \int_{\rho(x_0)}^{\rho(x)} \frac{d\rho}{\sqrt{2F(\rho)/G(\rho)}}\,; \quad F \equiv \int_{\rho_1}^{\rho} (p - \bar{p})\,\mathrm{E}(\rho)\,d\rho\,; \quad G \equiv \alpha\,\mathrm{E}(\rho)$$

– *Surface Tension:* $\sigma = \int_{-\infty}^{\infty} \left\{ \frac{1}{2}\left(T_{yy} + T_{zz}\right) - T_{xx} \right\} dx = \int_{-\infty}^{\infty} c\, \rho_x^2\, dx\,; \qquad c = \gamma' - \delta$

– *Statistical Models (D–S 1982):* $\gamma = 2\alpha\,, \quad \delta = 2\beta \quad \Rightarrow \quad c = \frac{2}{3}(a' - b)$

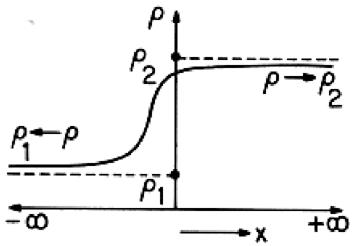
$$a = \frac{1}{16}\rho^2 u' + \frac{1}{2}\rho u\,, \quad b = \frac{1}{16}\rho^2 u'' + \frac{1}{4}\rho u' - \frac{1}{4}u\,, \quad c = \frac{1}{2}u + \frac{1}{4}\rho u'$$

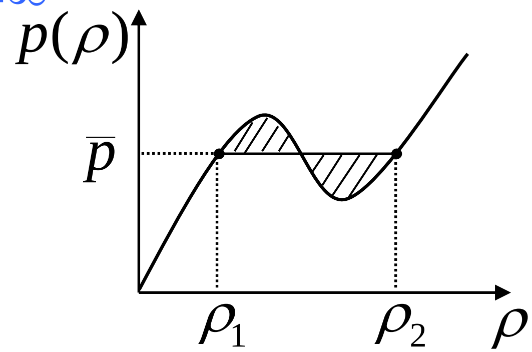
– *Validity of MR:* $\left(\frac{a}{\rho^2}\right)' = 2\left(\frac{b}{\rho^2}\right) \Rightarrow u = const.$

$$Exps\text{: } \mathrm{H_2O}\ \ \textbf{at}\ \ 100^0\ C\ \ \ldots \qquad \frac{\rho_2}{\rho_1} \to \begin{cases} \sim 1603.\,.\,.\ \text{Steam Tables} \\ \sim 16.\,.\,.\,.\,.\ MR \\ \sim 1660.\,.\,.\ \text{Our Theory} \end{cases}$$

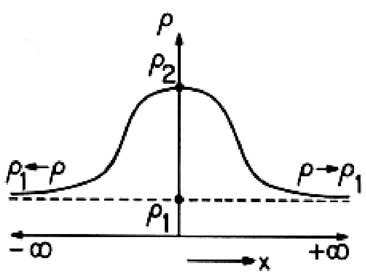
- ***Planar Interfaces / 1D Profiles***

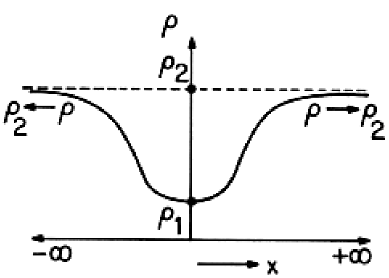
  - *Transitions (interfaces)* $\rho \to \rho_{1,2}$ *as* $x \to \mp\infty$

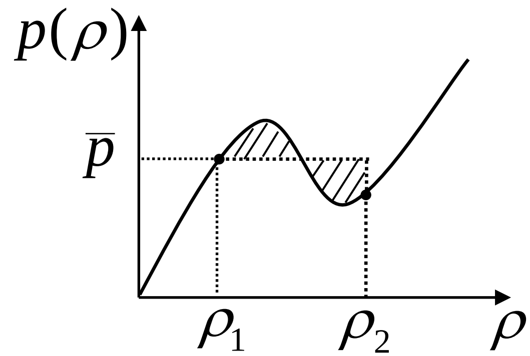




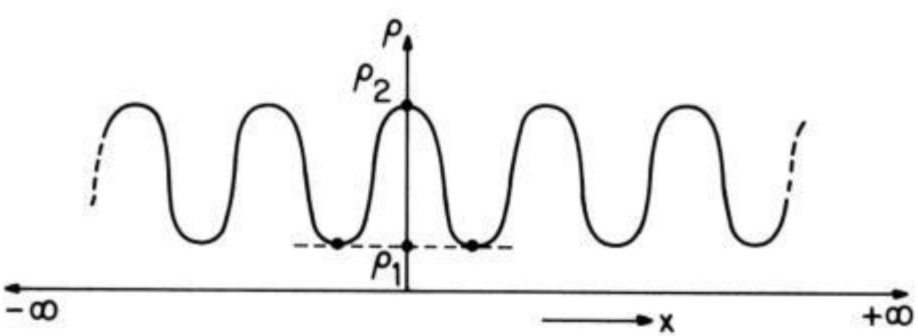
  - *Reversals (films)* $\rho \to \rho_1$ as $x \to \mp\infty$

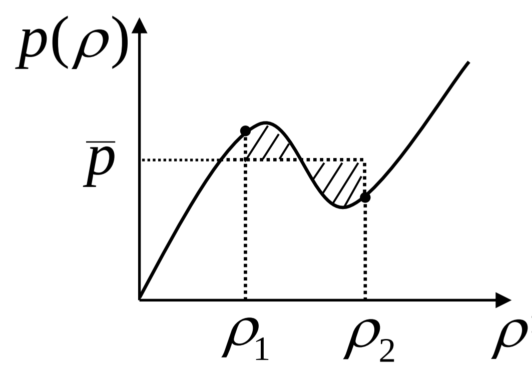






  - *Oscillations (layers)*

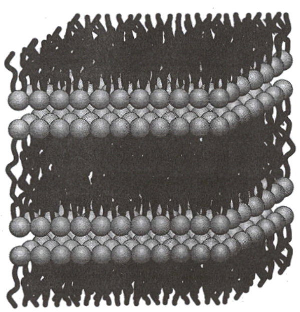




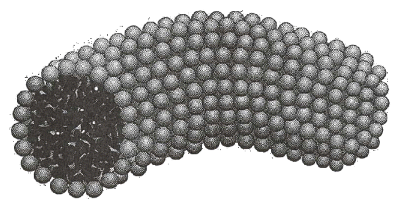
• ***General Interfaces / 3D Structures***

$$\nabla(-p + a\square\rho + \tilde{b}\left|\nabla\rho\right|^2) = (c\square\rho)\nabla\rho\ ; \qquad \begin{cases} \tilde{b} = b + \dfrac{1}{2}\left(c - a\dfrac{c'}{c}\right) \\ \square\rho \equiv \nabla^2\rho + \dfrac{1}{2}\dfrac{c'}{c}\left|\nabla\rho\right|^2 \end{cases}$$

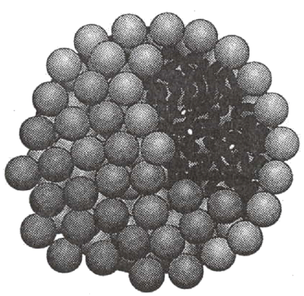
$$\tilde{b} \equiv 0 \Rightarrow MR \quad ; \quad \tilde{b} \neq 0 \Rightarrow \begin{cases} \rho = \rho(x) \\ \rho = \rho(r) \\ \rho = \rho(R) \end{cases}$$



layers



cylinders



spheres

Micelle Structures

Gradient Theories

Laplacian Regularization: A Unifying Ansatz

- ***Hooke's Law:*** $\boldsymbol{\sigma} = \lambda(tr\boldsymbol{\varepsilon})\mathbf{1} + 2G\boldsymbol{\varepsilon}$

$$\boldsymbol{\varepsilon} \to \frac{1}{V}\int_V G_{\varepsilon}(|\mathbf{r}-\mathbf{r}'|)\boldsymbol{\varepsilon}(\mathbf{r}')\,dV \quad \Rightarrow \quad \boldsymbol{\varepsilon} \to \boldsymbol{\varepsilon} - l_{\varepsilon}^2\nabla^2\boldsymbol{\varepsilon}$$

$$\therefore \quad \boldsymbol{\sigma} = \lambda(tr\boldsymbol{\varepsilon})\mathbf{1} + 2G\boldsymbol{\varepsilon} - c\nabla^2\left[\lambda(tr\boldsymbol{\varepsilon})\mathbf{1} + 2G\boldsymbol{\varepsilon}\right] \; ; \; c = l_{\varepsilon}^2$$

- ***Fick's Law:*** $\mathbf{j} = -D\nabla\rho$

$$\mathbf{j} \to \frac{1}{V}\int_V G_d(|\mathbf{r}-\mathbf{r}'|)\,\mathbf{j}(\mathbf{r}')\,dV \quad \Rightarrow \quad \mathbf{j} \to \mathbf{j} - l_d^2\nabla^2\mathbf{j}$$

$$\therefore \quad \dot{\rho} + div\mathbf{j} = 0 \;\Rightarrow\; \dot{\rho} = D\nabla^2\rho - c\nabla^4\rho \quad ; \quad c = l_d^2 D$$

- ***Von-Mises Flow:*** $\tau = \kappa(\gamma)\,;$ $\left[\begin{array}{l} \tau = \frac{1}{2}\sqrt{\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}'}\;;\;\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \frac{1}{3}(tr\boldsymbol{\sigma})\mathbf{1} \\ \gamma = \int \dot{\gamma}\,dt \;\;,\;\; \dot{\gamma} = \sqrt{2\dot{\boldsymbol{\varepsilon}}^p\cdot\dot{\boldsymbol{\varepsilon}}^p} \end{array}\right.$

$$\gamma \to \frac{1}{V}\int_V G_p(|\mathbf{r}-\mathbf{r}'|)\,\gamma(\mathbf{r}')\,dV \quad \Rightarrow \quad \gamma \to \gamma - l_p^2\nabla^2\gamma$$

$$\therefore \quad \tau = \kappa(\gamma) - c\nabla^2\gamma \;\; ; \; c = l_p^2\kappa'(\gamma)$$

❑ Note on Murray's Theory of Morphogenesis

[Gradient Bio-Chemo-Elasticity]

- ***The Crawling Cells***

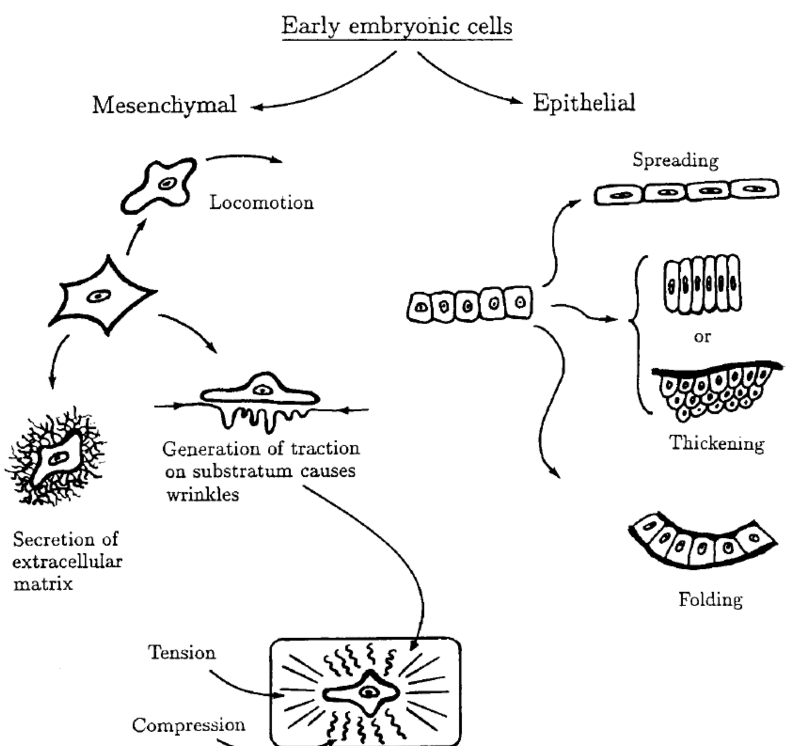


Figure 6.2. Early embryonic cells. Mesenchymal cells are motile, generate large traction forces and can secrete extracellular matrix which forms part of the tissue within which the cells move. When these cells are placed on a thin silicon rubber substratum their traction forces deform the rubber sheet; see the photograph in Figure 6.3. Epithelial cells do not move about but can spread or thicken when subjected to forces; this affects cell division (see, for example, Folkman and Moscona 1978).

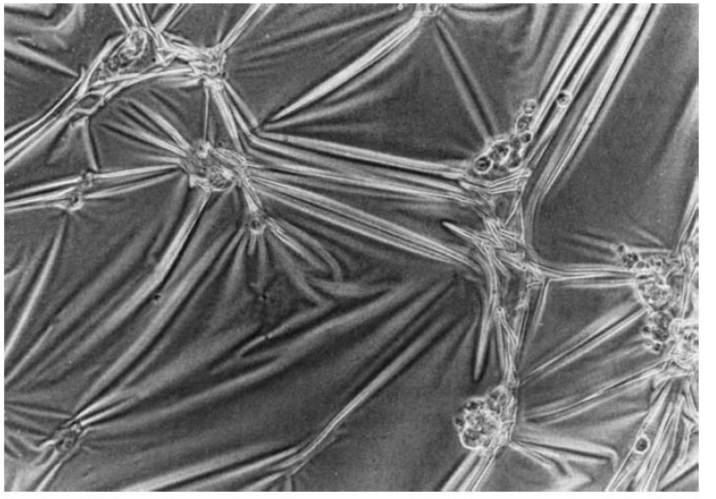


Figure 6.3. Mesenchymal cells on an elastic substratum. The strong tractions generated deform the substratum and create compression and tension wrinkles. The tension wrinkles can extend several hundreds of cell diameters. (Photograph courtesy of Albert K. Harris)

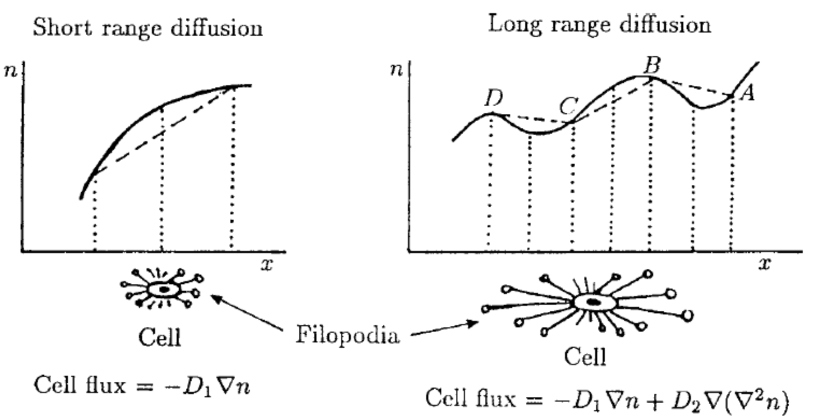


Figure 6.4. In (**a**) the filopodia only sense the immediate neighbouring densities to determine the gradient (the broken line) and hence disperse in a classical random manner giving a flux of $-D_1\nabla n$. With the situation in (**b**) the long filopodia can sense not only neighbouring densities but also neighbouring averages which contribute a long range diffusional flux term $D_2\nabla(\nabla^2 n)$. This contributes to directed dispersal which is not necessarily in the same direction as indicated by short range (again denoted by broken lines) diffusion. Long range diffusion suggests general movement of cells from A to D whereas short range diffusion implies movement from D to C, B to C and B to A.

- ***The Model Equations***
  - ***Cell Convection***

$$\mathbf{J}_C = n\frac{\partial \mathbf{u}}{\partial t} \quad ; \qquad \begin{cases} n \ldots \text{cell density} \\ \mathbf{u} \ldots \text{extracellular matrix (ECM) displacement} \end{cases}$$

  - ***Cell Dispersal***

$$\mathbf{J}_D = -D_1\nabla n + D_2\nabla\left(\nabla^2 n\right) \quad ; \qquad (D_1, D_2)\ldots \text{constants}$$

  - ***Cell Haptotaxis / Mechanotaxis***

$$\mathbf{J}_h = n\left(\alpha_1\nabla\rho - \alpha_2\nabla^3\rho\right) \quad ; \quad \rho \ldots \text{ECM density;} \quad \left(\alpha_1, \alpha_2\right)\ldots \text{constants}$$

  - ***Cell Conservation Law***

$$\frac{\partial n}{\partial t} + \mathrm{div}\left[\mathbf{J}_C + \mathbf{J}_D + \mathbf{J}_h\right] = M$$

$$M = r\,n\left(N - n\right) \quad \ldots \quad \text{mitosis}$$

$r \ldots$ rate constant, $N \ldots$ saturation constant

  - ***Cell / ECM Stresses***

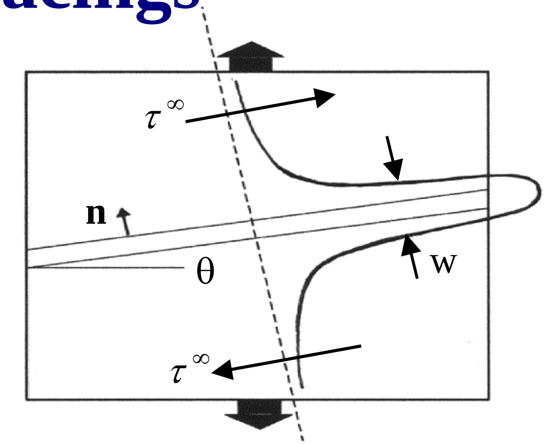
$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_{ECM} + \boldsymbol{\sigma}_{cell} \; ; \qquad \mathrm{div}\boldsymbol{\sigma} = 0$$

$$\boldsymbol{\sigma}_{ECM} = \left(\lambda\,\mathrm{tr}\boldsymbol{\varepsilon}\mathbf{1} + 2G\boldsymbol{\varepsilon}\right) - \ell^2\nabla^2\left(\lambda\,\mathrm{tr}\boldsymbol{\varepsilon}\mathbf{1} + 2G\boldsymbol{\varepsilon}\right); \quad \boldsymbol{\sigma}_{cell} = \frac{\tau\rho}{1+\beta n^2}\left(1+\gamma\nabla^2 n\right)\mathbf{1}$$

Gradient Plasticity

■ Capturing Shear Band Widths & Spacings

- ***Constitutive Equation***



$$\mathbf{S}' = -p\mathbf{1} + 2\mu\mathbf{D} \quad ; \quad \mathbf{D} \approx \dot{\boldsymbol{\varepsilon}}^{p}$$

$$\mu = \frac{\tau}{\dot{\gamma}} \quad , \quad \begin{cases} \tau \equiv \sqrt{\frac{1}{2}\mathbf{S}' \cdot \mathbf{S}'} \\ \dot{\gamma} \equiv \sqrt{2\mathbf{D} \cdot \mathbf{D}} \end{cases} ; \quad \tau = \kappa(\gamma) - c\nabla^{2}\gamma$$

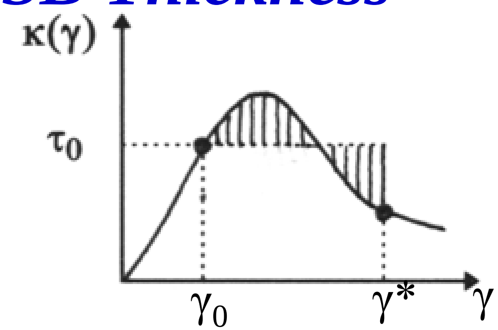
- ***Linear Stability / SB Orientation***

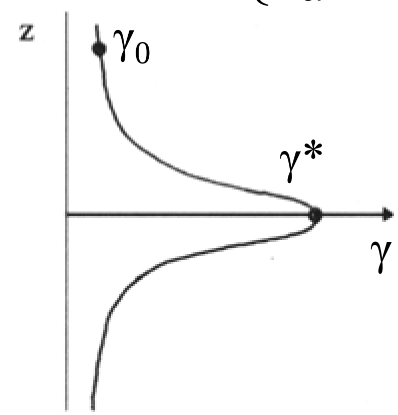
$$\mathrm{v} = \mathrm{L}_{\infty}\mathrm{x} + \tilde{\mathrm{v}}\mathrm{e}^{\mathrm{i}qz+\omega \mathrm{t}} ; \quad \omega > 0 \quad (\& \omega_{\max}) \rightarrow \theta_{cr} = \frac{\pi}{4} \quad \& \quad \begin{cases} h_{cr} = 0 \\ q_{cr} = 0 \end{cases}$$

- ***Nonlinear Solution / SB Thickness***

$$c\gamma_{zz} = \kappa(\gamma) - \tau_{0}$$

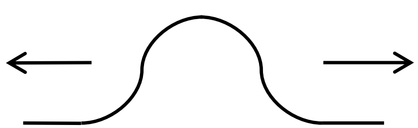
$$\gamma \equiv \int \dot{\gamma}\, dt$$





- ***Front Propagation***

Similar Procedure



Gradient Fluidity

■ Gradient Navier-Stokes/N-S

- ***Classical N-S (Incompressible)***

$$\mathbf{T} = -p\mathbf{1} + 2\mu\mathbf{D}; \quad \mathbf{D} = 1/2\,[grad\,\mathbf{v} + (grad\,\mathbf{v})^T]$$

$$div\mathbf{T} = \rho\dot{\mathbf{v}},$$

- ***Gradient N-S***

$$\mathbf{T} \to \mathbf{T} - \ell_T^2 \nabla^2 \mathbf{T}; \qquad \mathbf{D} \to \mathbf{D} - \ell_D^2 \nabla^2 \mathbf{D}$$

$$\therefore\ \mathbf{T} - \ell_T^2 \nabla^2 \mathbf{T} = -p\mathbf{1} + 2\mu\left(\mathbf{D} - \ell_D^2 \nabla^2 \mathbf{D}\right)$$

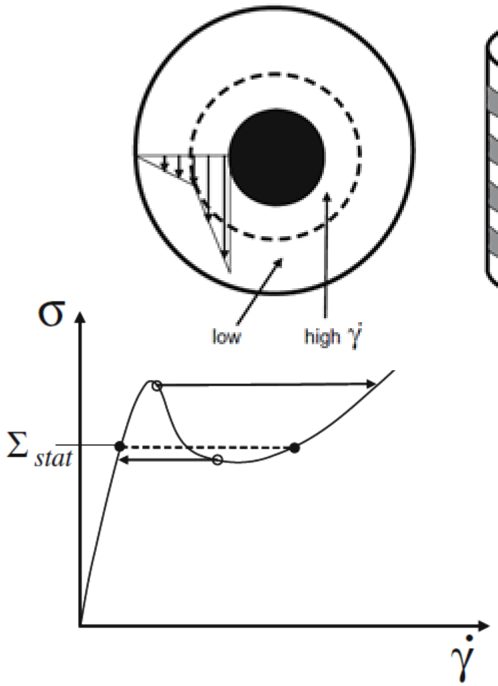
- ***Note 1:*** $\ell_T = 0;\ \Delta = \nabla^2,\ \Delta^2 = \nabla^4$

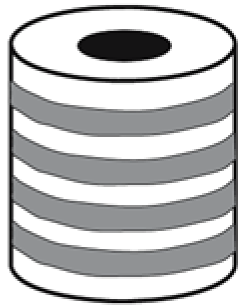
$$\therefore\ \rho\dot{\mathbf{v}} = -\nabla p + \mu\left(\Delta\mathbf{v} - \ell_D^2 \Delta^2 \mathbf{v}\right) \quad \text{Gurtin – Fried (2006)}$$

■ Gradient Rheology – Complex Fluids (Micelles)

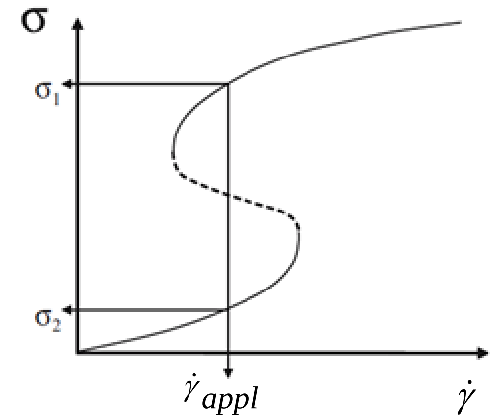
• *Experiments on Gradient & Vorticity Banding*

Gradient Banding Vorticity Banding





Schematic of the morphology of the patterns formed in gradient banding (left figure), where the inner cylinder is supposed to rotate, and vorticity banding (right figure) in a Couette geometry



The "van der Waals loop-like" behaviour of the standard shearstress σ, that is, the stress of the homogeneously sheared system, before banding occurred, as a function of the shear rate $\dot{\gamma}$. The *dashed horizontal* line marks the selected stress in the stationary state under controlled shear-rate conditions according to the "modified" equal area Maxwell construction. The horizontal arrows correspond to bottom and top jumps that are observed under controlled-stress conditions when the imposed stress is not equal to the modified equal-area stress

A necessary condition for the existence of a stationary vorticity banded state in case the shear rate is the same everywhere, independent of position, is that the shear-stress is multi-valued at the applied shear rate. In the stationary banded state, there are two types of microstructural order, corresponding to the two types of bands that can sustain different shear stresses σ_1 and σ_2 for the applied shear rate $\dot{\gamma}_{appl}$

• *Reviews: Olmsted et al; Yates et al; Dhont et al ~ 2000 – 2010*

- ***Gradient Models***

  - ***Diffusive Johnson-Segalman (DJS) Model***

$$\mathbf{T} = -p\mathbf{1} + 2\mu\mathbf{D} + \boldsymbol{\Sigma}$$

$$\overset{\mathbf{o}}{\boldsymbol{\Sigma}} + \frac{1}{\tau}\boldsymbol{\Sigma} = \begin{cases} \dfrac{2\mu^*}{\tau} + D\nabla^2\boldsymbol{\Sigma} & \dots\text{. Olmsted et al 2000} \\ \dfrac{2\mu^*}{\tau} - D\nabla^2\boldsymbol{\Sigma} & \dots\text{. Yuan 1999} \end{cases}$$

$$\overset{\mathbf{o}}{\boldsymbol{\Sigma}} = \left(\partial_t + \mathbf{v}\cdot\nabla\right)\boldsymbol{\Sigma} - (\boldsymbol{\Sigma}\mathbf{W} - \mathbf{W}\boldsymbol{\Sigma}) - \alpha\left(\mathbf{D}\boldsymbol{\Sigma} + \boldsymbol{\Sigma}\mathbf{D}\right)$$

  - ***Generalized Yuan et al Model 2010***

$$\mathbf{T} = 2\mu\mathbf{D} + \boldsymbol{\Sigma}$$

$$\frac{\partial\boldsymbol{\Sigma}}{\partial t} = f(\boldsymbol{\Sigma}, \mathbf{W}, \mathbf{D}) + \alpha_0\nabla^2\boldsymbol{\Sigma} + b_0\nabla^2\mathbf{D}$$

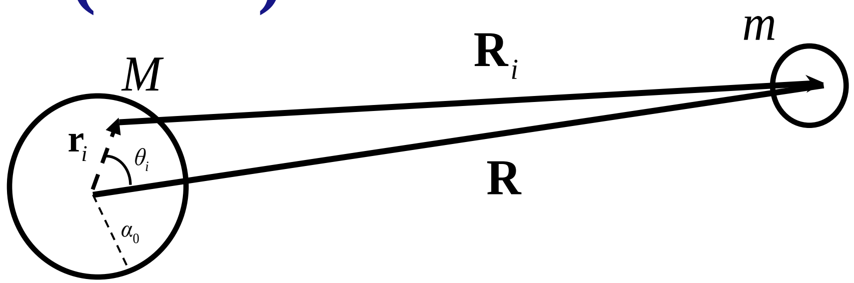
  - ***ECA Remarks***
    - Remark 1/ Gradient Vorticity: $\mathbf{W} \to \mathbf{W} - \ell_W^2\nabla^2\mathbf{W}$
    - Remark 2/ Corotational Derivative: $\boldsymbol{\omega} = \mathbf{W} - \mathbf{W}^p$

$\mathbf{W}^p$ ~ plastic spin-like of crystal plasticity

Gradients in Fundamental Forces

■ Gradient Gravitational Force (GGF)

- ***Newton's Law***



$$\boldsymbol{F} = \frac{GM\,m}{R^3}\boldsymbol{R} = \frac{GM\,m}{R^2}\boldsymbol{e_R};\quad \boldsymbol{e_R} = \frac{\boldsymbol{R}}{R}$$

- M Distributed $(0 < r < \alpha_0)$/ m Point Mass

$$f_i(\mathbf{r}) = \int G_{ij}(\mathbf{r}-\mathbf{r}')F_j(\mathbf{r}')d^3\mathbf{r}';\; G_{ij}(\mathbf{r}-\mathbf{r}') : \text{Nonlocal Interaction Kernel}$$

- ***Fourier Transform/Taylor Expansion***

[redacted]

$$\tilde{G}_{ij}(\mathbf{k}) \approx G_{ij}(0) + \alpha k^2 \delta_{ij},\; \alpha = d^2 G_{ij}(0)/dk^2,\; k = |\mathbf{k}|$$

- ***FT Inversion***

$$\mathbf{f} = \left(1 - \ell^2 \nabla^2\right)\mathbf{F},\; \ell^2 = \mathrm{sgn}(\alpha)\left|d^2 G_{ij}(0)/dk^2\right|$$

- ***Governing Equation/Radial Symmetry***

$$\left(1-\ell^2\nabla^2\right)\mathbf{f}=\mathbf{F}; \quad \mathbf{f}=F(r)\mathbf{e}_r;\ \mathbf{F}=F_N(r)\mathbf{e}_r, F_N=\frac{A}{r^2}$$

$$F-\ell^2\left(\frac{\partial^2 F}{\partial r^2}+\frac{2}{r}\frac{\partial F}{\partial r}-\frac{2F}{r^2}\right)=\frac{A}{r^2}$$

- ***Solution*** *(F* $\rightarrow$ *0 for* r $\rightarrow$ ∞*)*

$$F=\frac{A}{r^2}\left[1+Be^{-r/\ell}\left(1+\frac{r}{\ell}\right)\right]$$

- ***Properties***
  - $r\rightarrow 0 \Rightarrow F=F_{SF}=\frac{AB}{r^2}\left(1-\frac{r^2}{2\ell^2}\right)\approx\frac{AB}{r^2}$ ... Strong Force?
  - $r\rightarrow\infty \Rightarrow F=F_N\approx\frac{A}{r^2}$ .... Newton Force

- ***Note 1:*** *Behavior of* F_{SF} **:** $F_{SF}/F_N\approx B;\ B>>1\Rightarrow F_{SF}>>F_N$

- ***Note 2: Behavior of r vs ℓ***

  - ***For*** $\ell \sim r \Rightarrow F = \frac{A}{r^2}(1+\frac{2B}{e})$; for $F = F_{SF} \approx \frac{\hbar c}{r^2} \Rightarrow B \approx \frac{\hbar c}{A}$
  - ***For*** $r << \ell \Rightarrow F = \frac{A}{r^2}(1+B(1-\frac{r^2}{2\ell^2})) \approx \frac{AB}{r^2} \approx \frac{\hbar c}{r^2} \Rightarrow B = \frac{\hbar c}{A}$
  - ***For*** $r >> \ell \Rightarrow F = \frac{A}{r^2}(1+Be^{-r/\ell}(1+\frac{r}{\ell})) \approx \frac{A}{r^2} \Rightarrow B$ undetermined

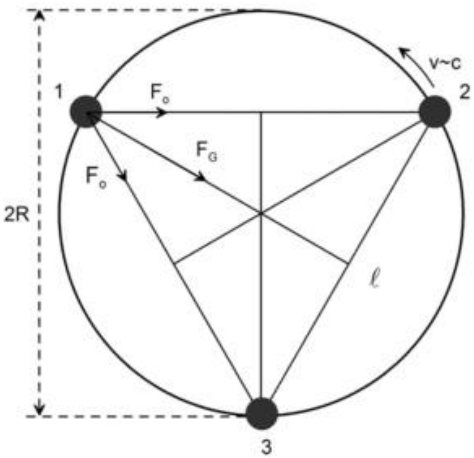
- ***Identification of Model/Configuration Parameter ℓ***

  - ***De Broglie/Relativistic*** $\ell = \hbar / \gamma m_0 c \approx 6.31\times 10^{-16} m$
  - ***Compton*** $\ell = \hbar / m_p c \approx 2.10\times 10^{-16} m$
  - ***Planck*** $\ell = \sqrt{\hbar G / c^3} = 1.61\times 10^{-35} m$
  - ***Schwarzschild*** $\begin{cases} \text{microscopic black hole (mBH)}\ \ell = 2G m_p / c^2 = 4.48\times 10^{-54} m \\ \text{supermassive black hole (SMBH)}\ \ell = 2G m_{BH} / c^2 = 5.0\times 10^{11} m \end{cases}$

m_p: proton mass, m_0 : neutrino mass, m_{BH} : SMBH mass, $\gamma = 1/\sqrt{1-v^2/c^2}$

- ***Identification of Model Parameter B***

[Assume Vayenas 2012 RNM Configuration]



– ***De Broglie length:*** $\ell = \lambda = \hbar / (\gamma m_0 v)$

– ***Centrifugal Force:*** $F_C = \gamma m_0 v^2 / r \approx \gamma m_0 c^2 / r$

– ***Relativistic Neutrino Mass:*** $m_\nu = 3m = 3\gamma m_0$

– ***Proton Energy:*** $m_p c^2 = 3\gamma m_0 c^2 \rightarrow \gamma = m_p / 3m_0 = 7.82 \times 10^9$

– ***GGF:*** $F = \dfrac{A}{\sqrt{3r^2}}(1 + Be^{-r/\ell}(1 + \dfrac{r}{\ell})) = F_C = \dfrac{\gamma m_0 c^2}{r}; \quad A = G m_0^2 \gamma^2$

– ***For*** $r = \ell = 6.32 \times 10^{-16} m \Rightarrow B = \dfrac{\sqrt{3} e \gamma m_0 c^2 \lambda}{2A} = \dfrac{\sqrt{3} e \hbar c}{2 G m_0^2 \gamma^2} = 3.58 \times 10^{39}$

$\therefore \; F = 7.06 \times 10^4 \text{ N}$

– ***Note: Vayenas Model*** $F = \dfrac{G m_0^2 \gamma^6}{\sqrt{3r^2}} = F_C = \dfrac{\hbar c}{r^2}$

$$\gamma = 3^{1/12} m_{Pl}^{1/3} m_0^{-1/3} = 7.38 \times 10^9, \lambda = \frac{\hbar}{\gamma m_0 c} = 6.69 \cdot 10^{-16} \text{m}, \; F = 7.06 \times 10^4 \text{ N}$$

■ Gradient Intermolecular Forces (GIF)

– ***Van Der Waals (1875):*** $(P+\frac{a}{V^2})(V-b)=RT$

– ***London (1930) Quantum Mechanics (QM):*** $F=-\frac{dw}{dr}, w(r)=-\frac{C}{r^6}$

$$w(r)=\begin{cases} -\frac{3\alpha_0^2 h\nu}{4(4\pi\varepsilon_0)^2}\frac{1}{r^6}=-\frac{C}{r^6}; & r\geq\sigma \\ \infty \quad ; & r<\sigma \end{cases}$$

$$\therefore a=\frac{2\pi}{3}C\sigma^{-3}, b=\frac{2\pi}{3}\sigma^3$$

$$\sigma = \text{molecular size}$$

- ***Other Examples***

– ***Yukawa/Screened Coulomb:*** $\pm\frac{A}{r}e^{-r/r_0}$

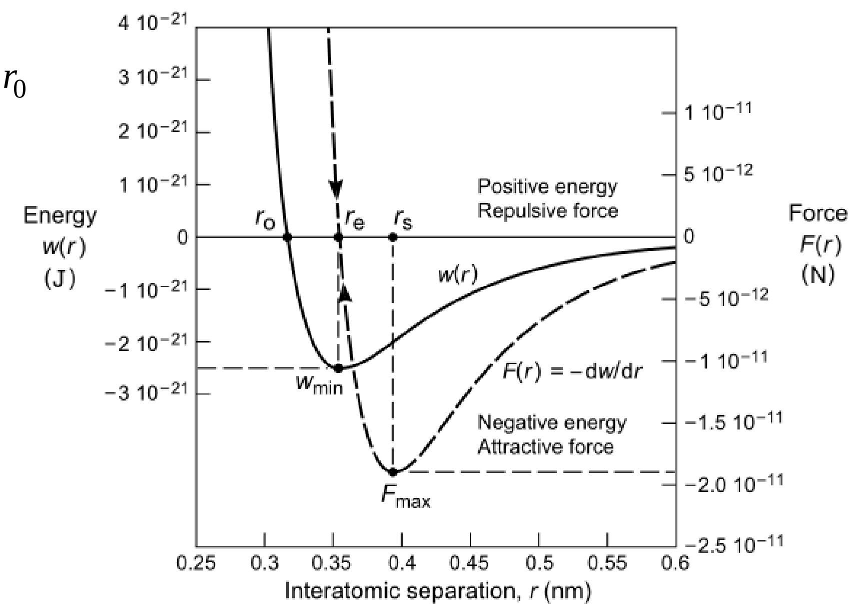
– ***Mie:*** $-\frac{A}{r^n}+\frac{B}{r^m}$

– ***Lennard-Jones:*** $-\frac{A}{r^6}+\frac{B}{r^{12}}$

– ***Buckingham:*** $-\frac{A}{r^6}+Be^{-r/r_0}$

– ***Morse:*** $-Ae^{-r/r_A}+Be^{-r/r_B}$

– ***Parabolic/Elastic:*** $A(r_0-r)^2; F=\pm 2A(r-r_0)$



Gradient Van der Waals/London Force

- ***Governing Equation***

$$\left(1-\ell^2\nabla^2\right)\varphi=\varphi_0,\quad \varphi_0(r)=w(r)=-\frac{C}{r^6},\ F=-\frac{d\varphi}{dr}$$

- ***Solution*** **(**$\varphi\to 0$ for $r\to\infty$**)**; A.... integration constant

$$\varphi(r)=A\ell\frac{e^{-r/\ell}}{r}-\frac{C}{48}\left\{\frac{4\ell^4}{r^4}+\frac{2\ell^2}{r^2}+\frac{\ell}{r}\left[e^{r/\ell}\,\mathrm{Ei}\left(-\frac{r}{\ell}\right)-e^{-r/\ell}\,\mathrm{Ei}\left(\frac{r}{\ell}\right)\right]\right\}$$

$$\mathrm{Ei}(x)=\int_{-x}^{\infty}\frac{e^{-t}}{t}dt;\quad \varphi(r)\to\left(\frac{\ell}{r}\right)^4 \text{ for } \ell>>r\to 0;\quad \varphi(r)\to-\frac{C}{r^6}\text{ for } r>>\ell$$